Math Virtual Learning

Precalculus with Trigonometry

May 14, 2020

Precalculus with Trigonometry
Lesson: May 14th, 2020

Objective/Learning Target:
Students will be introduced to the Trigonometric Form of a Complex Number and will be able to multiply and divide the trig form of complex numbers.

Let's Get Started:

Watch the video below to see an introduction to the Trig Form of Complex Numbers.

Watch Video: Trigonometric Form of Complex Numbers

OPTIONAL VIDEO: Trigonometric Form of a Complex Number
(This video is just another similar introduction to Trig Form of Complex Numbers.)

Trigonometric Form of a Complex Number

The **trigonometric form** of the complex number $z = a + bi$ is

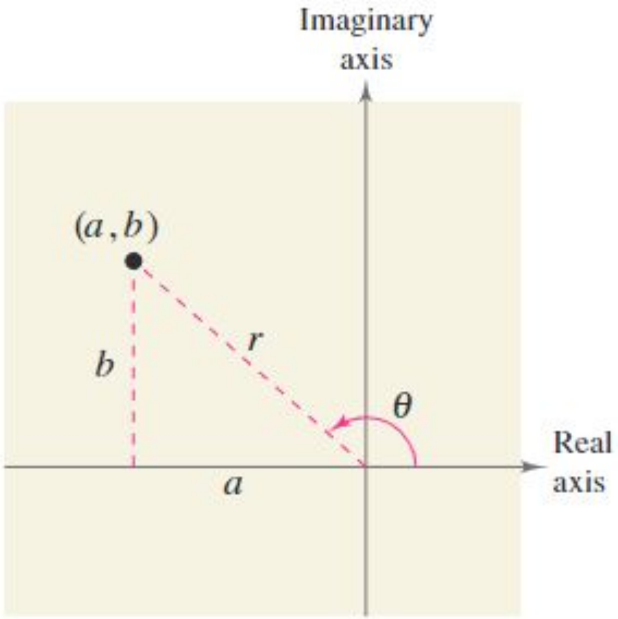
$$z = r(\cos\theta + i\sin\theta)$$

where $a = r\cos\theta$, $b = r\sin\theta$, $r = \sqrt{a^2 + b^2}$, and $\tan\theta = b/a$. The number r is the **modulus** of z, and θ is called an **argument** of z.

Definition of the Absolute Value of a Complex Number

The **absolute value** of the complex number $z = a + bi$ is

$$|a + bi| = \sqrt{a^2 + b^2}.$$



Product and Quotient of Two Complex Numbers

Let $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ and $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$ be complex numbers.

$$z_1 z_2 = r_1 r_2[\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)]$$ Product

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)], \quad z_2 \neq 0$$ Quotient

Now What??????

What can be done with complex numbers in trig form?

Video: Multiply and Divide Complex Numbers in Trigonometric Form (Formulas)

Example #1:

Plot $z = -2 + 5i$ and find its absolute value.

Solution

The number is plotted in Figure 6.45. It has an absolute value of

$$|z| = \sqrt{(-2)^2 + 5^2}$$
$$= \sqrt{29}.$$

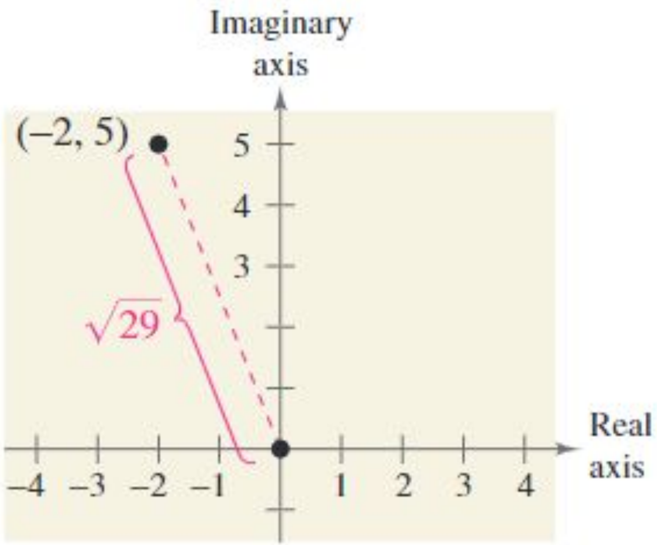


FIGURE 6.45

Example #2:

Write the complex number $z = -2 - 2\sqrt{3}i$ in trigonometric form.

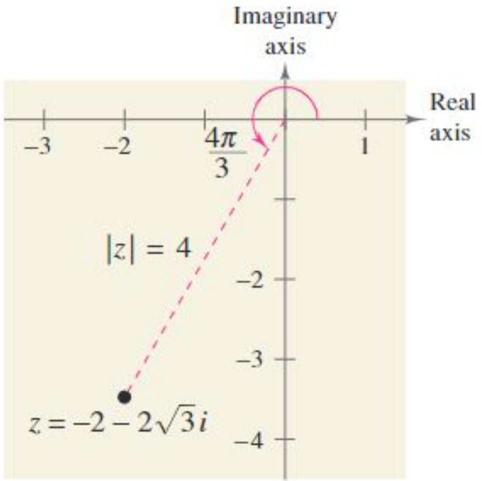


FIGURE 6.47

Solution

The absolute value of z is

$$r = \left|-2 - 2\sqrt{3}i\right| = \sqrt{(-2)^2 + \left(-2\sqrt{3}\right)^2} = \sqrt{16} = 4$$

and the reference angle θ' is given by

$$\tan\theta' = \frac{b}{a} = \frac{-2\sqrt{3}}{-2} = \sqrt{3}.$$

Because $\tan(\pi/3) = \sqrt{3}$ and because $z = -2 - 2\sqrt{3}i$ lies in Quadrant III, you choose θ to be $\theta = \pi + \pi/3 = 4\pi/3$. So, the trigonometric form is

$$z = r(\cos\theta + i\sin\theta)$$

$$= 4\left(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right).$$

See Figure 6.47.

Example #3:

Write the complex number in standard form $a + bi$.

$$z = \sqrt{8}\left[\cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)\right]$$

Solution

Because $\cos\left(-\frac{\pi}{3}\right) = \frac{1}{2}$ and $\sin\left(-\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$, you can write

$$\begin{aligned} z &= \sqrt{8}\left[\cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)\right] \\ &= 2\sqrt{2}\left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) \\ &= \sqrt{2} - \sqrt{6}i. \end{aligned}$$

Example #4:

Find the product z_1z_2 of the complex numbers.

$$z_1 = 2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right) \qquad z_2 = 8\left(\cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6}\right)$$

Solution

$$\begin{aligned}
z_1z_2 &= 2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right) \cdot 8\left(\cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6}\right) \\
&= 16\left[\cos\left(\frac{2\pi}{3} + \frac{11\pi}{6}\right) + i\sin\left(\frac{2\pi}{3} + \frac{11\pi}{6}\right)\right] \\
&= 16\left(\cos\frac{5\pi}{2} + i\sin\frac{5\pi}{2}\right) \\
&= 16\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right) \\
&= 16[0 + i(1)] \\
&= 16i
\end{aligned}$$

Multiply moduli and add arguments.

You can check this result by first converting the complex numbers to the standard forms $z_1 = -1 + \sqrt{3}i$ and $z_2 = 4\sqrt{3} - 4i$ and then multiplying algebraically, as in Section 2.4.

$$\begin{aligned}
z_1z_2 &= (-1 + \sqrt{3}i)(4\sqrt{3} - 4i) \\
&= -4\sqrt{3} + 4i + 12i + 4\sqrt{3} \\
&= 16i
\end{aligned}$$

Example #5:

Find the quotient z_1/z_2 of the complex numbers.

$$z_1 = 24(\cos 300^\circ + i \sin 300^\circ) \qquad z_2 = 8(\cos 75^\circ + i \sin 75^\circ)$$

Solution

$$\frac{z_1}{z_2} = \frac{24(\cos 300^\circ + i \sin 300^\circ)}{8(\cos 75^\circ + i \sin 75^\circ)}$$

$$= \frac{24}{8}[\cos(300^\circ - 75^\circ) + i \sin(300^\circ - 75^\circ)]$$

Divide moduli and subtract arguments.

$$= 3(\cos 225^\circ + i \sin 225^\circ)$$

$$= 3\left[\left(-\frac{\sqrt{2}}{2}\right) + i\left(-\frac{\sqrt{2}}{2}\right)\right]$$

$$= -\frac{3\sqrt{2}}{2} - \frac{3\sqrt{2}}{2}i$$

Practice

On a separate piece of paper, do each of the following problems. Answers will be provided on the next page.

1. Write $-1+\sqrt{3}\,i$ in trigonometric form.

2. Write $2(\cos 60° + i \sin 60°)$ in standard form.

3. Multiply $\left[2\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)\right]\left[6\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)\right]$

4. Divide $\dfrac{12(\cos 92° + i\sin 92°)}{2(\cos 122° + i\sin 122°)}$

Practice - **ANSWERS**

On a separate piece of paper, do each of the following problems. Answers will be provided on the next page.

1. Write $-1+\sqrt{3}\,i$ in trigonometric form.
2. Write $2(\cos 60° + i\sin 60°)$ in standard form.
3. Multiply $\left[2\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)\right]\left[6\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)\right]$
4. Divide $\dfrac{12(\cos 92° + i\sin 92°)}{2(\cos 122° + i\sin 122°)}$

1. $2(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3})$

2. $1+\sqrt{3}i$ or $1+i\sqrt{3}$

3. $12(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3})$

4. $6(\cos 330° + i\sin 330°)$

Additional Resource Videos:

Multiplying Complex Numbers in Trig Form

Multiplying Complex Numbers In Trigonometric Form

Additional Practice:

Complex Numbers Rectangular and Trig Form

Problems 1-18

Please note that when it asks for Polar Form, it means Trig Form

Practice for Trig and Complex Form